Find $\lim_{x\to\infty} \coth x$ **algebraically**. SCORE: ______ / 3 PTS

$$\underbrace{\lim_{x\to\infty} \frac{e^{2x}+1}{e^{2x}-1}}_{1} = \underbrace{\lim_{x\to\infty} \frac{2\cancel{e^{2x}}}{\cancel{2}e^{2x}}}_{} = \underbrace{1}_{1}$$

$\frac{\infty}{\infty}$

L'HÔPITAL'S RULE 1

For this question, you may use the formulae for $\frac{d}{dx}\sinh x$, $\frac{d}{dx}\cosh x$ and/or $\frac{d}{dx}\tanh x$ without proving them. **SCORE: ______ / 8 PTS**

If you need to use the formula for the derivative of any other hyperbolic function, you must prove it.

[a] **Without using the exponential formula for sech x,** prove the formula for $\frac{d}{dx}\text{sech}\, x$.

$$\frac{d}{dx}\frac{1}{\cosh x} = -(\cosh x)^{-2}\sinh x \quad 2$$

$$= -\frac{1}{\cosh x}\frac{\sinh x}{\cosh x}$$

$$= -\text{sech}\, x \tanh x \quad 1$$

[b] **Without using the logarithmic formula for $\tanh^{-1} x$,** prove the formula for $\frac{d}{dx}\tanh^{-1} x$.

$$y = \tanh^{-1} x$$

$$x = \tanh y \quad 1$$

$$1 = \text{sech}^2 y \frac{dy}{dx} \quad 1\tfrac{1}{2}$$

$$1 = (1-\tanh^2 y)\frac{dy}{dx}$$

$$1 = (1-x^2)\frac{dy}{dx}$$

$$\cosh^2 y - \sinh^2 y = 1$$

$$1 \quad 1 - \tanh^2 y = \text{sech}^2 y$$

$$\frac{dy}{dx} = \frac{1}{1-x^2} \quad 1\tfrac{1}{2}$$

Find $\frac{d}{dx} x^2 \cosh^{-1}(x^5)$. Simplify your final answer as a single fraction.

SCORE: ______ / 4 PTS

You may use the derivatives of any hyperbolic or inverse hyperbolic functions from your textbook without proving them.

$$2x \cosh^{-1} x^5 + x^2 \frac{5x^4}{\sqrt{(x^5)^2 - 1}}$$

$$= \underbrace{2x \cosh^{-1} x^5}_{1} + \frac{\underbrace{5x^6}_{1}}{\underbrace{\sqrt{x^{10} - 1}}_{2}}$$

If $\coth x = -5$, find $\sinh x$. ALTERNATE SOLUTION ON NEXT PAGE

SCORE: ______ / 4 PTS

$\cosh^2 x - \sinh^2 x = 1$

$\coth^2 x - 1 = \operatorname{csch}^2 x$ 1

$24 = \operatorname{csch}^2 x$ 1

$\operatorname{csch} x = \cancel{\pm} 2\sqrt{6} \longrightarrow$

$= -2\sqrt{6}$ $\frac{1}{2}$

$\sinh x = -\dfrac{1}{2\sqrt{6}} = -\dfrac{\sqrt{6}}{12}$

$\frac{1}{2}$ $\frac{1}{2}$

$\coth x = \dfrac{\cosh x}{\sinh x}$

SINCE $\coth x < 0$

AND $\cosh x > 0$ (FOR ALL x),

$\sinh x < 0$

SO $\operatorname{csch} x < 0$ $\frac{1}{2}$

If $\coth x = -5$, find $\sinh x$.

SCORE: ______ / 4 PTS

$\frac{1}{2}$ POINT EACH

$$\underbrace{\tanh x = -\frac{1}{5}}$$

$$\frac{\sinh x}{\cosh x} = -\frac{1}{5}$$

$$\underbrace{\sinh x = -\frac{1}{5}\cosh x}$$

$$= \underbrace{-\frac{1}{2\sqrt{6}}} = \underbrace{-\frac{\sqrt{6}}{12}}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\underbrace{1 - \tanh^2 x = \operatorname{sech}^2 x}$$

$$\underbrace{\frac{24}{25} = \operatorname{sech}^2 x}$$

$$\underbrace{\frac{2\sqrt{6}}{5} = \operatorname{sech} x} > 0$$

$$\underbrace{\frac{5}{2\sqrt{6}} = \cosh x}$$

Prove the logarithmic formula for $\sinh^{-1} x$ given in your textbook. SCORE: ______ / 5 PTS

NOTE: This is NOT a question about derivatives.

$y = \sinh^{-1} x$

1 $x = \sinh y = \frac{e^y - e^{-y}}{2}$

LET $z = e^y$, SO $z > 0$ $\frac{1}{2}$

$\frac{1}{2}$ $x = \frac{z - \frac{1}{z}}{2}$

$2x = z - \frac{1}{z}$

$2xz = z^2 - 1$

$\frac{1}{2}$ $0 = z^2 - 2xz - 1$

1 $z = \frac{2x \pm \sqrt{4x^2+4}}{2} = x \pm \sqrt{x^2+1}$ $\frac{1}{2}$

$e^y = z = x + \sqrt{x^2+1}$ $\frac{1}{2}$

$\sinh^{-1} x = y = \ln(x + \sqrt{x^2+1})$ $\frac{1}{2}$